Computed Fluid Dynamics: Basic Concepts and Rationale

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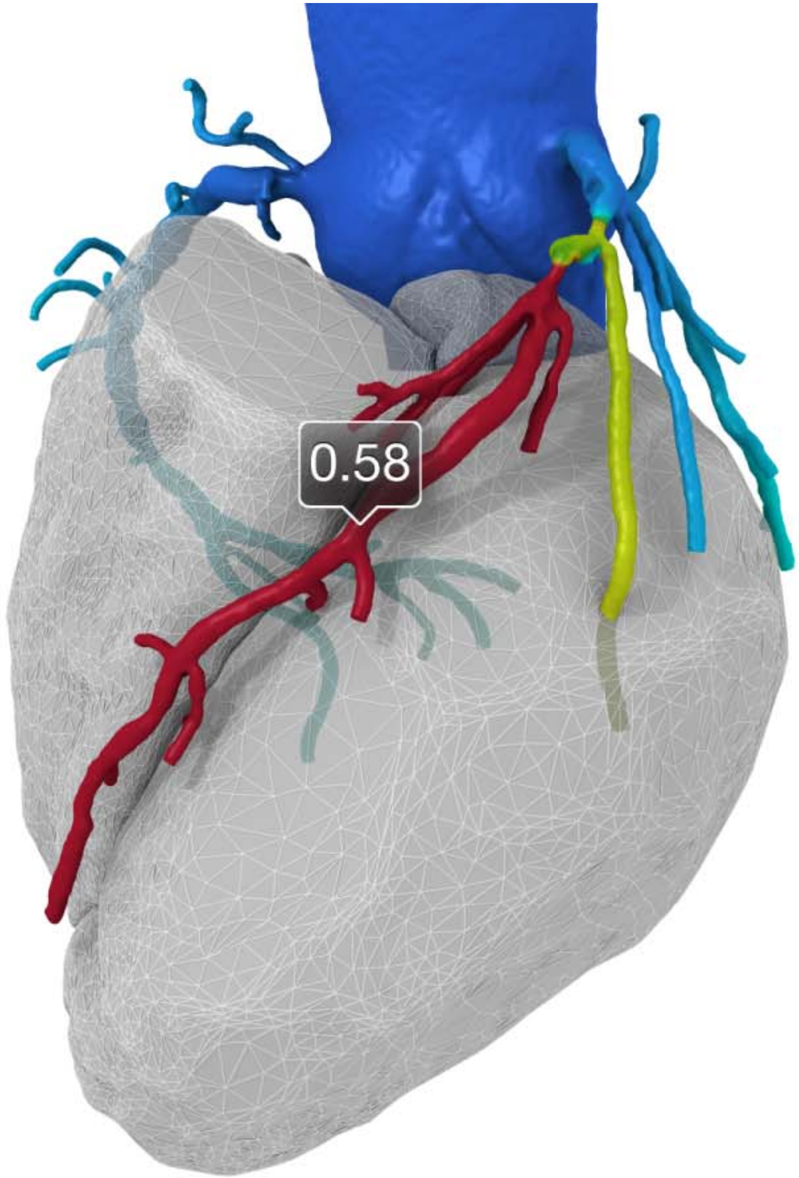

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Disclosures

- Founder, Shareholder and Employee of HeartFlow, Inc.

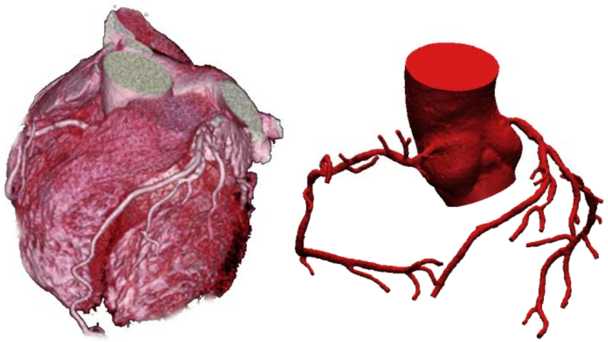
HeartFlow™

simulation

HeartFlow Process for Obtaining FFR_{CT}

Computational Model based on coronary CTA

3-D quantitative, anatomic model from coronary CTA



Physiologic models:

- Myocardial demand
- Morphometry-based boundary conditions
- Effect of adenosine on microcirculation

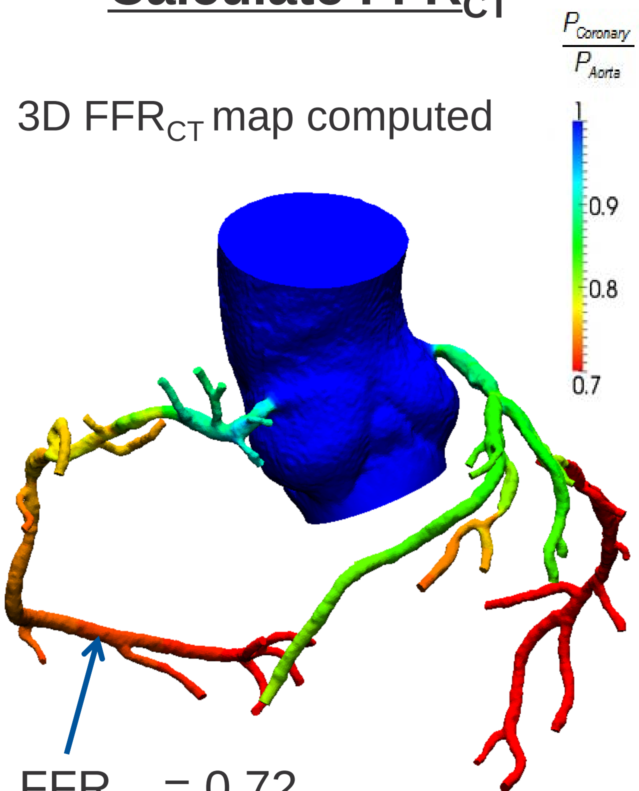
Blood Flow Solution

Blood flow equations solved on supercomputer

$$\rho \bar{v}_{,t} + \rho \bar{v} \cdot \nabla \bar{v} = -\nabla p + \nabla \cdot \underset{\sim}{\tau}$$
$$\nabla \cdot \bar{v} = 0$$

Calculate FFR_{CT}

3D FFR_{CT} map computed



FFR_{CT} = 0.72
(can select any point on model)

Q&A

Q: How do we compute FFR?

A: By solving the governing equations of blood flow for pressure and velocity fields on a patient-specific geometric model subject to appropriate physiologic boundary conditions.

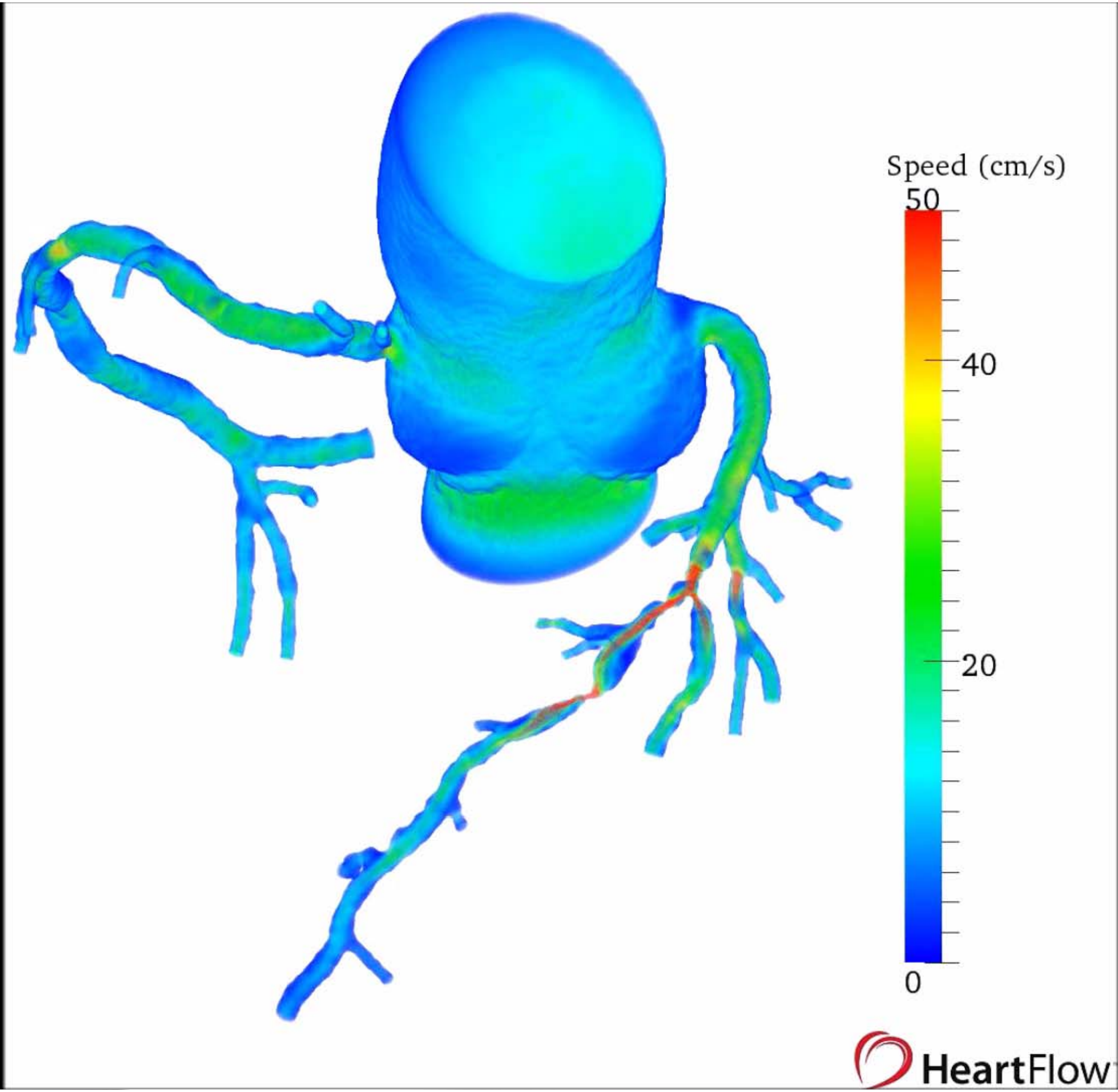
Q: What are the governing equations of blood flow?

A: The governing equations of blood flow are the equations of mass conservation and momentum balance. These equations are solved for the unknown pressure which is a function of 3 spatial coordinates and time, i.e. $p=p(x,y,z,t)$ and for the three components of blood velocity v_x, v_y, v_z which are each functions of position and time, i.e.

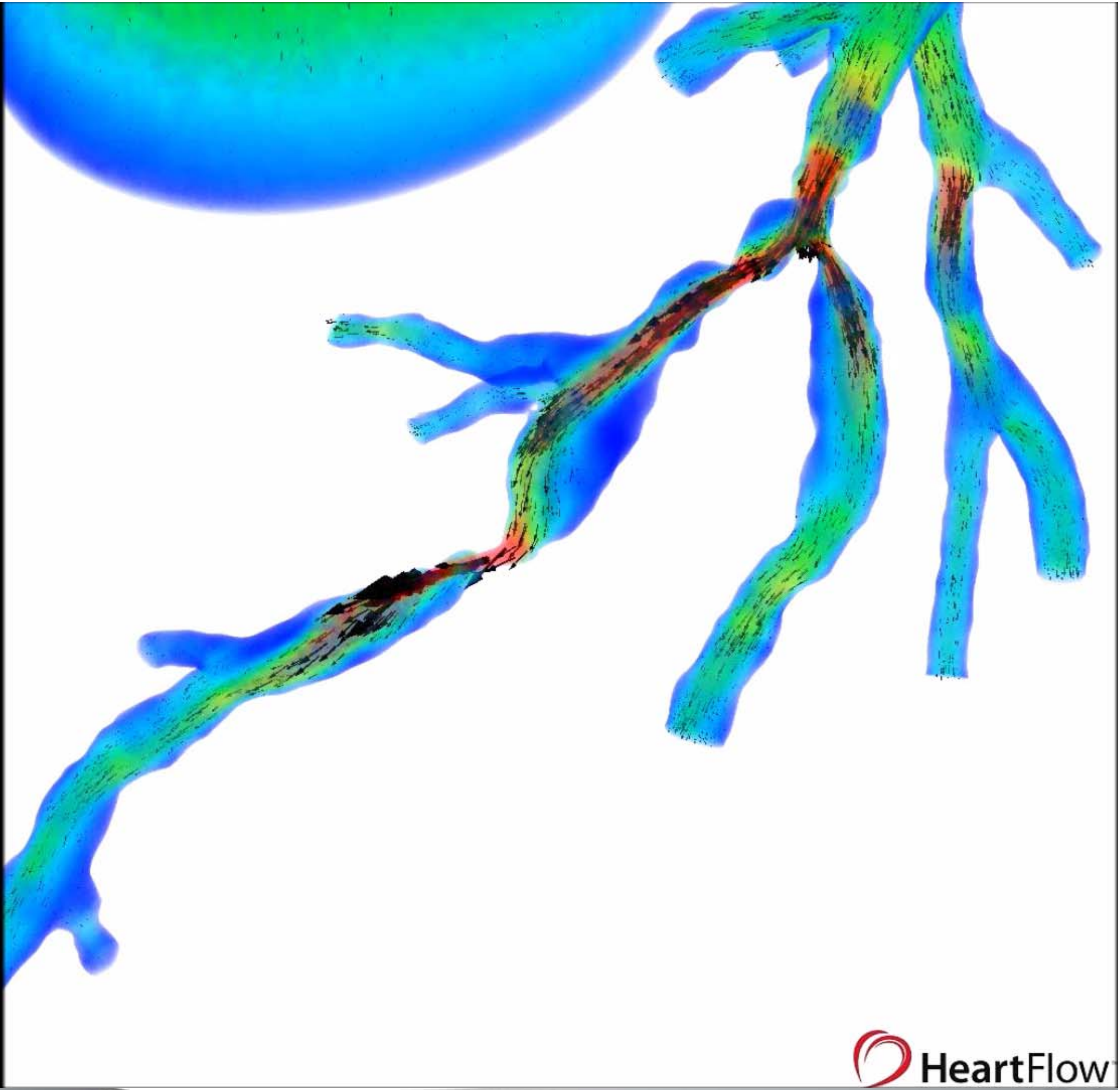
$$v_x = v_x(x,y,z,t)$$

$$v_y = v_y(x,y,z,t)$$

$$v_z = v_z(x,y,z,t)$$



Speed (cm/s)
50
40
20
0
HeartFlow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Governing Equations of Blood Flow

Mass Conservation (1 equation):

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}=0$$

This law states that blood is an incompressible fluid

Momentum Balance (3 equations):

$$\rho\frac{\partial v_x}{\partial t}+\rho\left(v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)$$

$$\rho\frac{\partial v_y}{\partial t}+\rho\left(v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)$$

$$\rho\frac{\partial v_z}{\partial t}+\rho\left(v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)$$

These equations come from the application of Newton's 2nd law, F=ma to a fluid

where ρ is the fluid density, and μ is the fluid viscosity (both assumed known).

We solve these for $v_x(x,y,z,t), v_y(x,y,z,t), v_z(x,y,z,t), p(x,y,z,t)$

for every point in the 3D model and over whatever time interval we are interested in.

Q&A

Q: Do the properties of blood matter?
A: Yes, we need to specify the viscosity and density of blood. Blood density is virtually constant, viscosity can be calculated from Hematocrit.

Q: Why do we need a computer to solve these equations?
A: The governing equations of blood flow are highly nonlinear and the geometry of the patient-specific model is very complicated.

Q&A

Q: How do we solve the equations of blood flow?
A: We use a *numerical method* to approximate the governing equations and obtain an approximate solution at a finite (but very large) number of points. These methods for solving fluid flow problems are called computational fluid dynamics (CFD) methods.

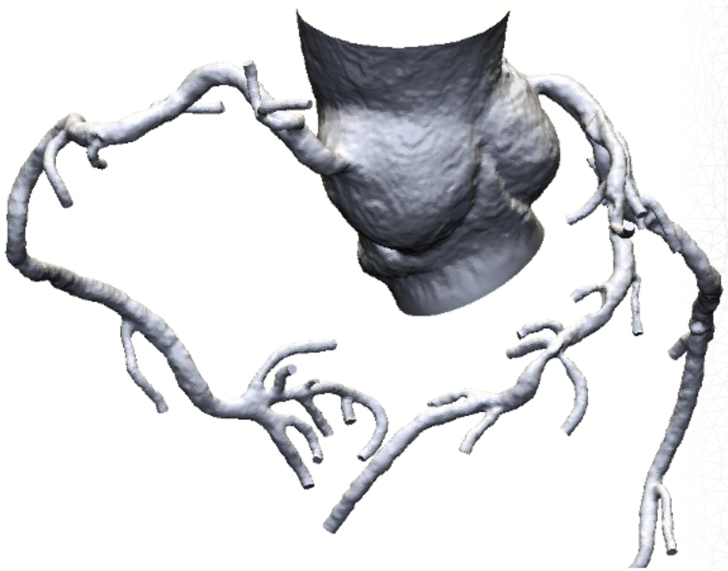
Q: What type of computational fluid dynamics methods are typically used to solve blood flow equations?
A: The CFD method most commonly used is called the *finite element method*.

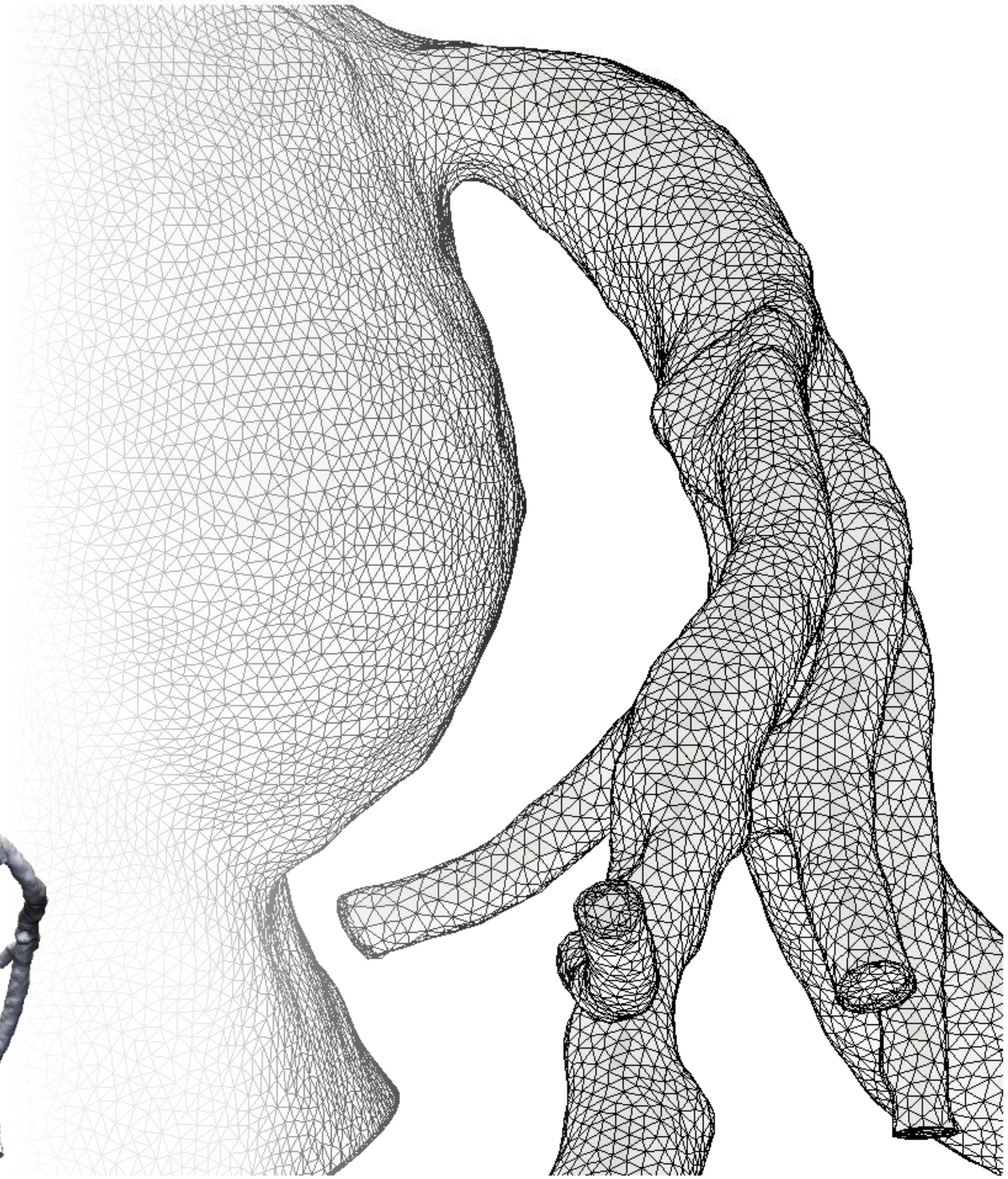
Q: How does the finite element method work?
A1: We approximate the geometry using a *finite element mesh* consisting of millions of *nodes* and millions of *elements*.
A2. We then solve the governing equations at the nodes which are distributed throughout the model. This requires solving millions of nonlinear equations simultaneously and repeating this process for thousands of time intervals in a cardiac cycle.

The finite element mesh

The elements are linear tetrahedrons filling the volume. The faces of these elements are the triangles we see in the image to the right.

The nodes are the vertices of the tetrahedron.





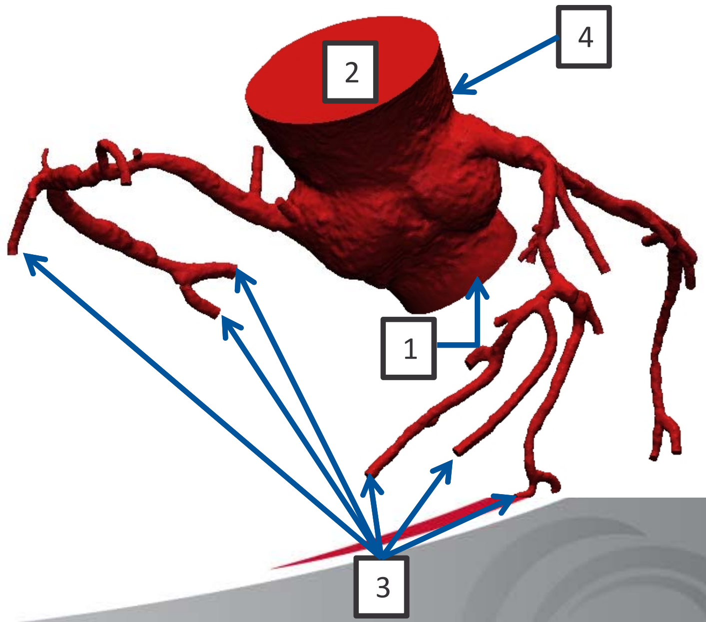
Q&A

Q: What are "appropriate physiologic boundary conditions"?

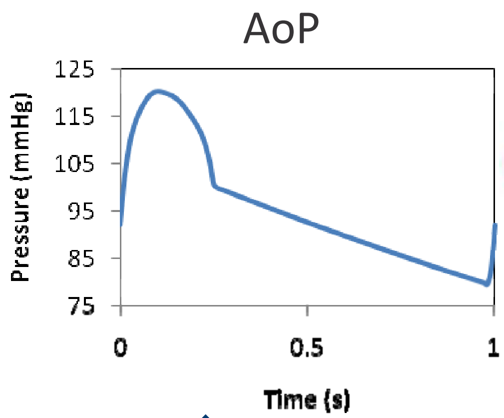
A: Since we are only solving the equations of blood flow in a portion of the patient's circulatory system, we have to say something about our variables of interest (pressure and velocity) at the interface between the modeled domain and the remainder of the circulation.

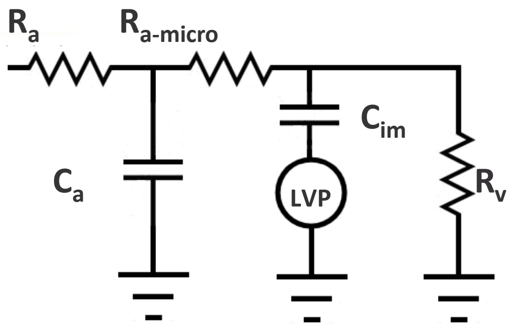
There are four types of boundaries we have to consider and each has a distinct boundary condition. These are:

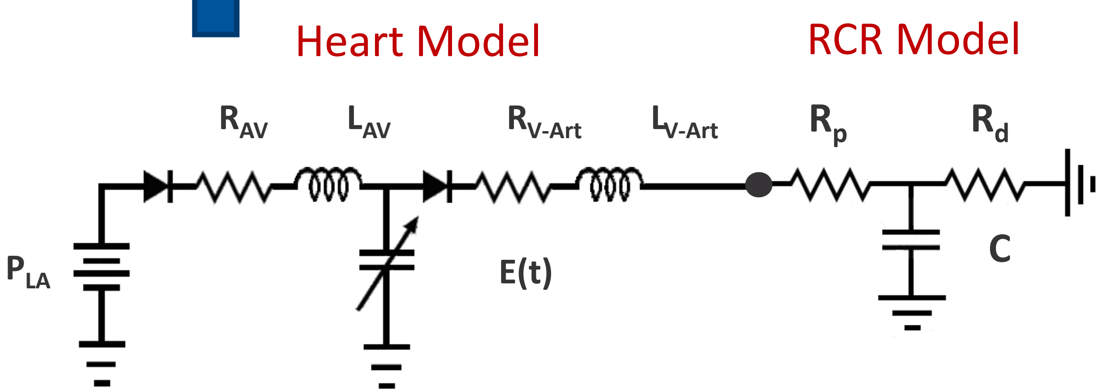
1. Aortic inlet – we model the interactions between the LV and systemic circulation
2. Aortic outlet – we enforce a relationship between flow and pressure of form representing aortic impedance.
3. Coronary outlets – we will prescribe a unique model of the microcirculation at each and every coronary outlet boundary.
4. Lateral surface – we can neglect the motion of the arteries and prescribe a zero velocity ("no-slip condition" for viscous fluids).

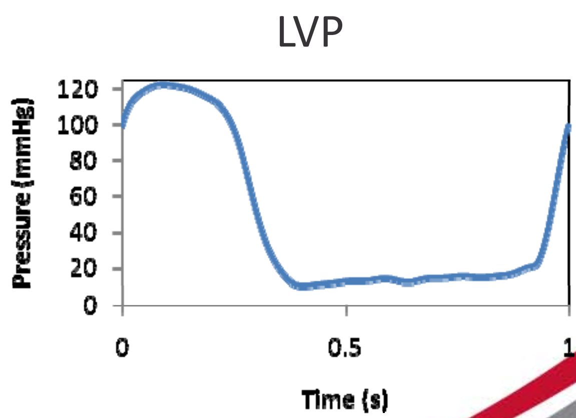


Idealized stenosis – model, boundary conditions

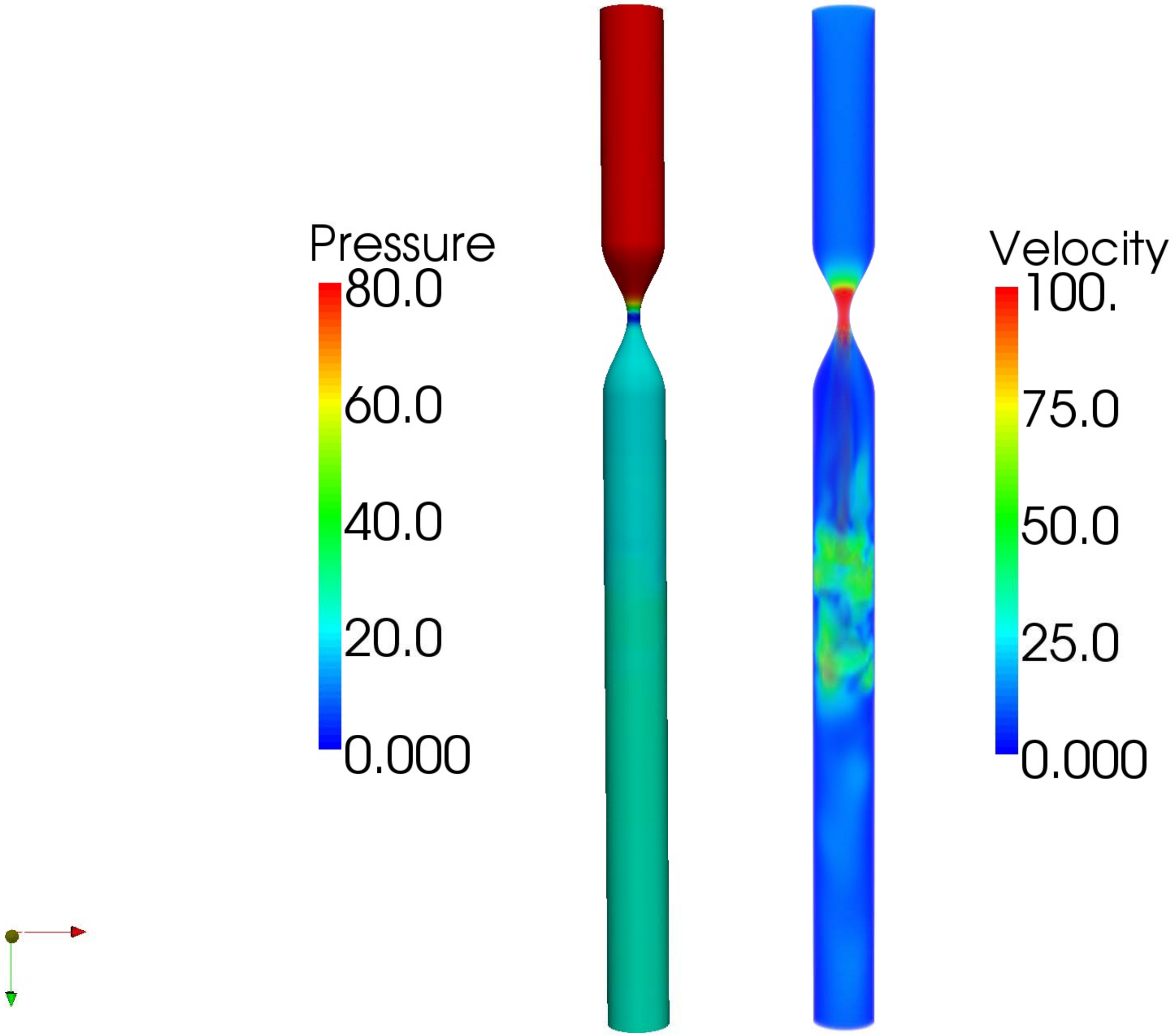




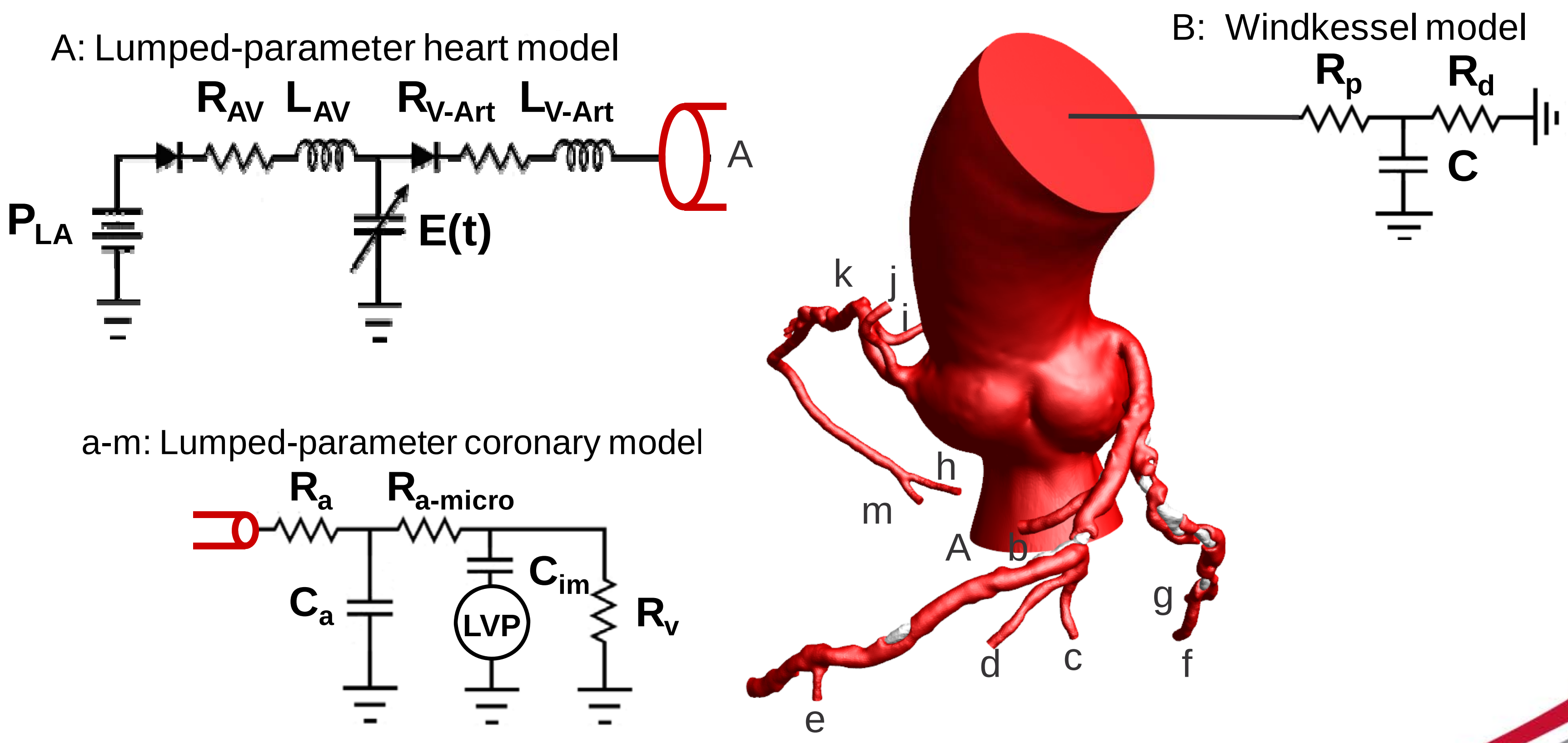




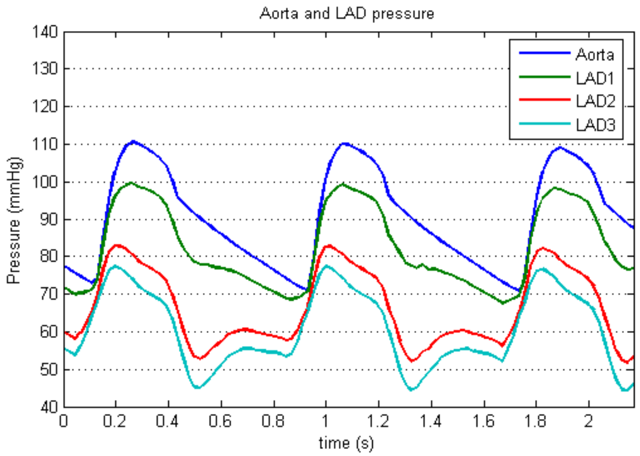
Pressure and velocity in 80% area reduction stenosis under hyperemic conditions

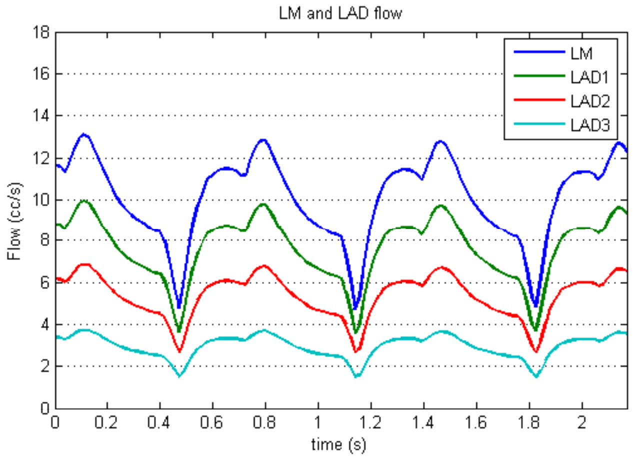


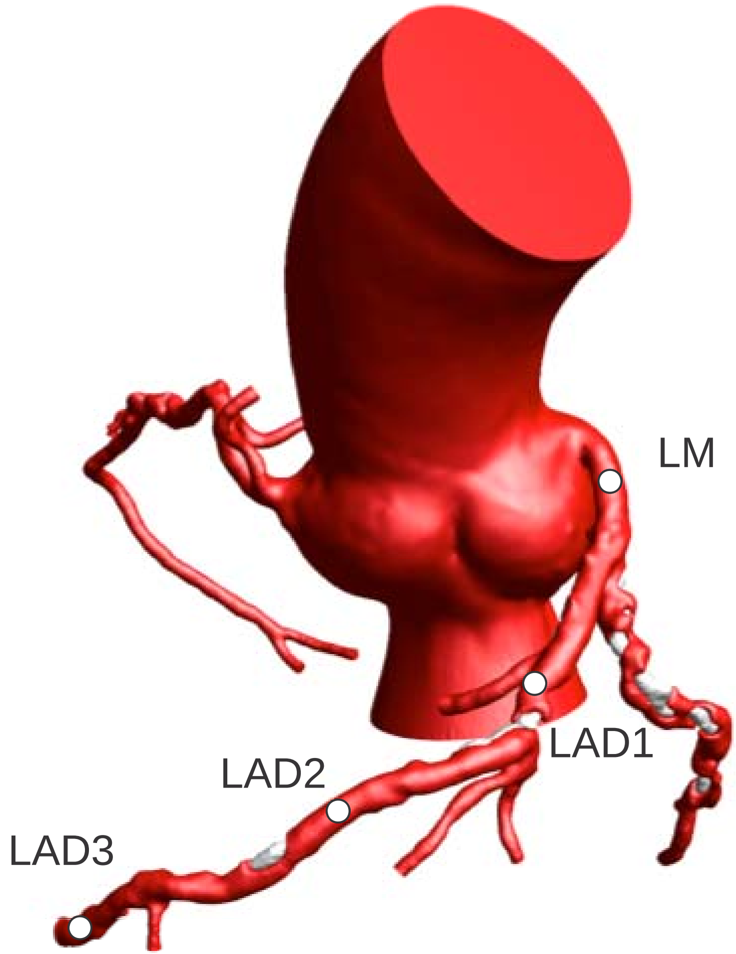
Patient-Specific model of aorta and coronary arteries



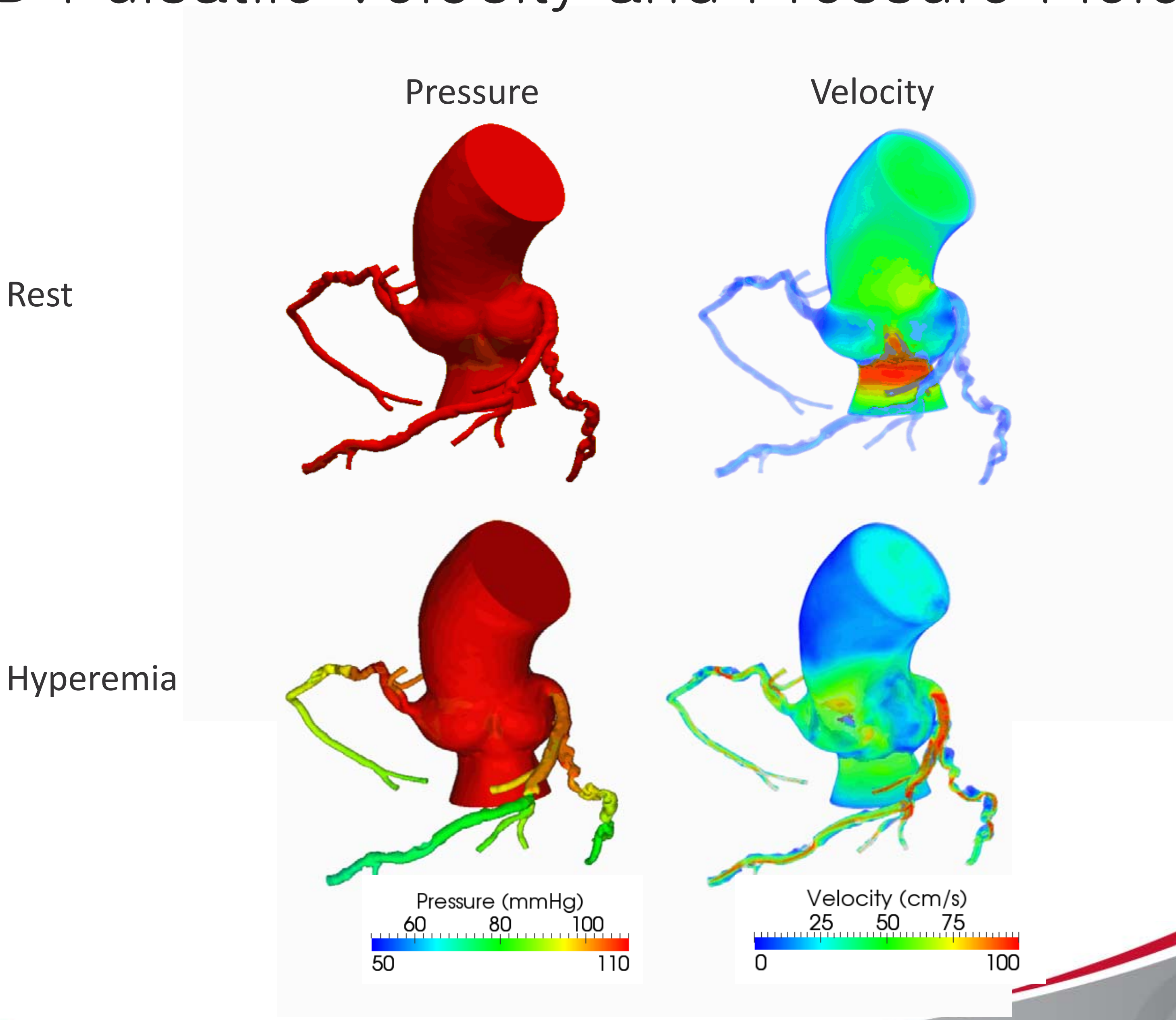
Calculated blood flow & pressure waveforms during hyperemia





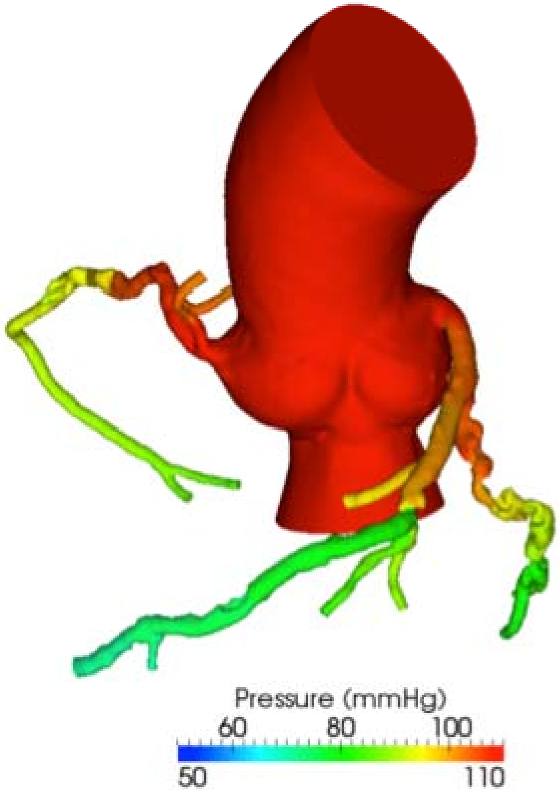


3D Pulsatile Velocity and Pressure Fields

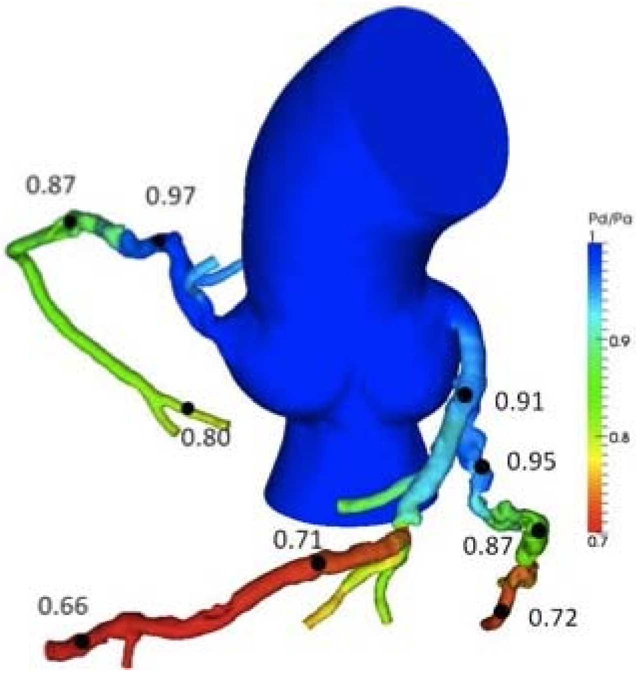


From Hyperemic Pressure to FFR_{CT}

- Mean coronary pressure is divided by aortic pressure in hyperemic state to compute FFR



Simulated mean pressure with hyperemia



FFR = Coronary / aortic pressure with hyperemia

Summary

- CFD methods enable the solution of a broad range of fluid dynamics problems governed by laws of physics
- CFD methods yield approximate solutions to mathematical models
- Cardiovascular fluid dynamics problems present unique challenges especially in creating patient-specific models and assigning realistic boundary conditions
- There are innumerable applications of CFD technology in the cardiovascular system

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Thank you